Schemes 2018 Exercise 4

Question 1. Let $< f_i >$ be a system of polynomial equations defined over an algebraically closed field F. Let $F \subseteq \Omega$ for Ω some field. Prove that $< f_i >$ has solution in Ω iff it has solution in F.

Question 2. Prove that $dim(A) = dim(spec(A))$.

Question 3. Prove that $dim(A) \geq dim(A/I)$ and $dim(A) \geq dim(S^{-1}A)$ for every ideal I of A and every multiplicative system S in A.

Question 4. Prove that if X is Noetherian space and $X = \cup_{i=1}^{n} Z_i$, Z_i closed in X, then $dim(X) = max_i dim(Z_i)$.

Question 5. For a topological space X and a point $x \in X$, define its local dimension at x, denoted $dim_x(X)$, to be the maximal length of a chain of irreducible closed subsets containing x. Prove that if A is an integral domain which is a finitely generated algebra over a field F, then $dim_x(spec(A)) = dim(spec(A))$ for every point $x \in spec_m(A)$.

Question 6.

- Let R be a Noetherian ring of finite Krull dimension. Let $f(x) \in R[x]$ be a on-zero polynomial. Prove that $dim(R[x]/(f(x))) \leq dim(R)$.
- Use the previous result to prove that $dim(R[x]) = dim(R) + 1$.